KERNEL MATCHING: CONTINUOUS SHAPE MATCHING WITHOUT ISOMETRY ASSUMPTIONS

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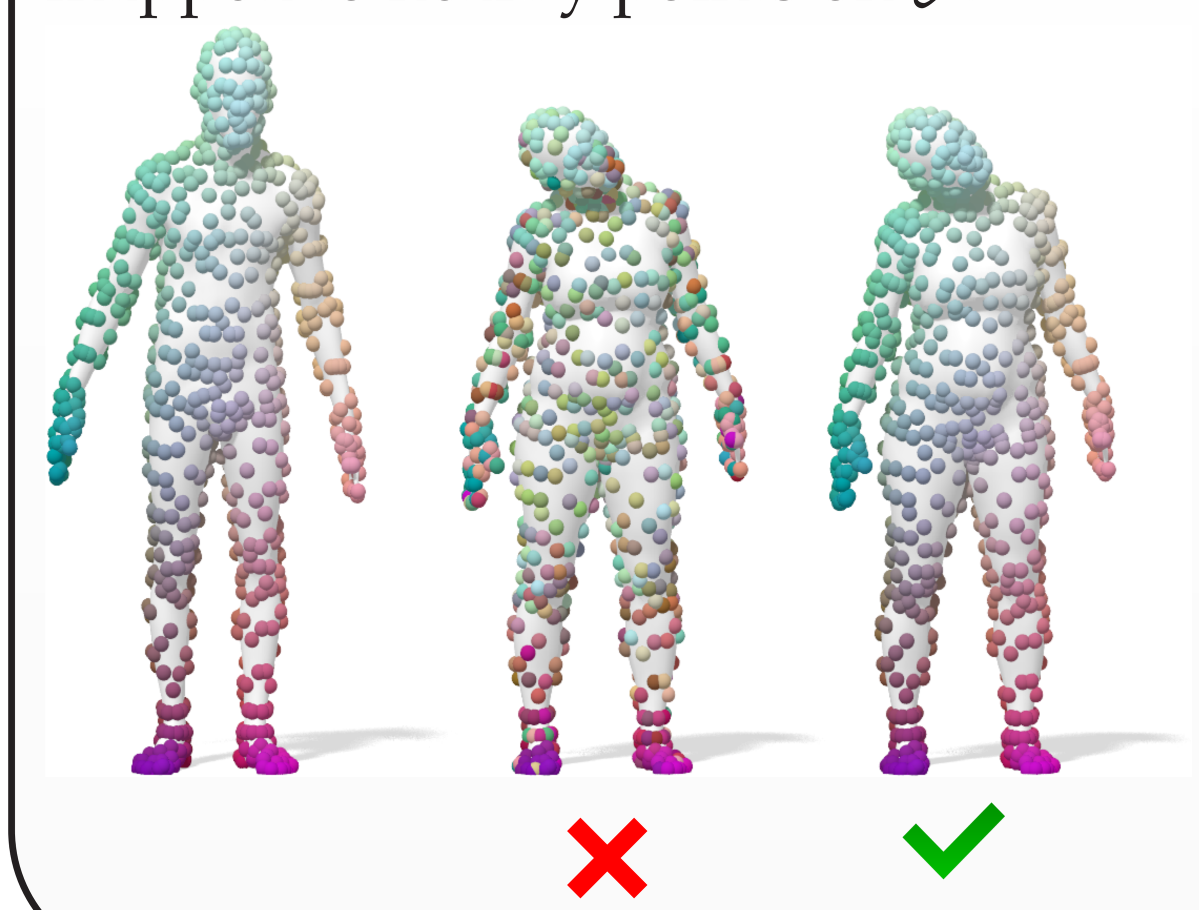
Abstract

We present a method for finding correspondence between two manifolds based on matching their heat kernels. Despite the combinatorial nature of the problem, we provide a simple algorithm with an appealing interpretation.

Continuous Shape Matching

Find correspondence between shapes $\mathcal{X}, \mathcal{Y}$ such that nearby points on $\mathcal{X}$ are mapped to nearby points on $\mathcal{Y}$.



Descriptor Matching

Our method falls into the category of descriptor matching.

Pointwise descriptors [2, 1]
$\mathbf{F}_{\mathcal{X}} \in \mathbb{R}^{n_{\mathcal{X}} \times k}, \mathbf{F}_{\mathcal{Y}} \in \mathbb{R}^{n_{\mathcal{Y}} \times k}$

Pairwise descriptors [3, 5, 4]
$\mathbf{K}_{\mathcal{X}} \in \mathbb{R}^{n_{\mathcal{X}} \times n_{\mathcal{X}}}, \mathbf{K}_{\mathcal{Y}} \in \mathbb{R}^{n_{\mathcal{Y}} \times n_{\mathcal{Y}}}$

References

[1] Nombari, F., Salti, S., Di Stefano, L, *Unique signatures of histograms for local surface description*, *European Conference on Computer Vision (pp. 356-369)*, 2010.

[2] Rustamov, R. M. *Laplace-Beltrami eigenfunctions for deformation invariant shape representation*, *Proceedings of the fifth Eurographics symposium on Geometry processing (pp. 225-233)*. .Eurographics Association, 2007.

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Kernel Matching

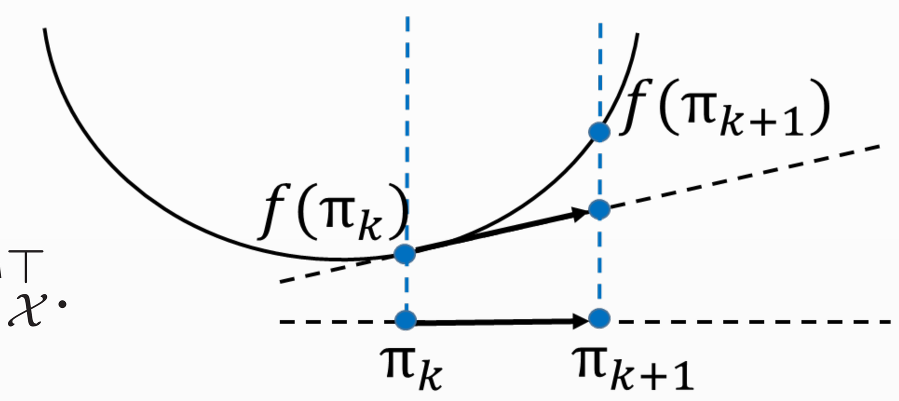
We model the problem as a maximization over a function $f(\mathbf{\Pi})$ consisting of a data fitting term (descriptor matching) plus a regularization term promoting the continuity of the map $\mathbf{\Pi}$ (kernel matching).

$$\max_{\mathbf{\Pi}} \operatorname{tr}\left(\mathbf{\Pi}^\top \mathbf{F}_{\mathcal{Y}} \mathbf{F}_{\mathcal{X}}^\top\right) + \lambda \operatorname{tr}\left(\mathbf{\Pi}^\top \mathbf{K}_{\mathcal{Y}}^\top \mathbf{\Pi} \mathbf{K}_{\mathcal{X}}\right) \quad \text{s.t.} \quad \begin{cases} \mathbf{\Pi}\mathbf{1} \leq \mathbf{1} \\ \mathbf{\Pi}^\top \mathbf{1} = \mathbf{1} \\ \mathbf{\Pi} \in \{0,1\}^{n_{\mathcal{Y}} \times n_{\mathcal{X}}} \end{cases}$$

We iteratively solve the following *linear assignment problem*:

$$\mathbf{\Pi}_{k+1} = \arg \max_{\mathbf{\Pi} \in \mathcal{P}_{n_{\mathcal{X}}}^{n_{\mathcal{Y}}}} \operatorname{tr}\left(\mathbf{\Pi}^\top \nabla f(\mathbf{\Pi}_k)\right)$$

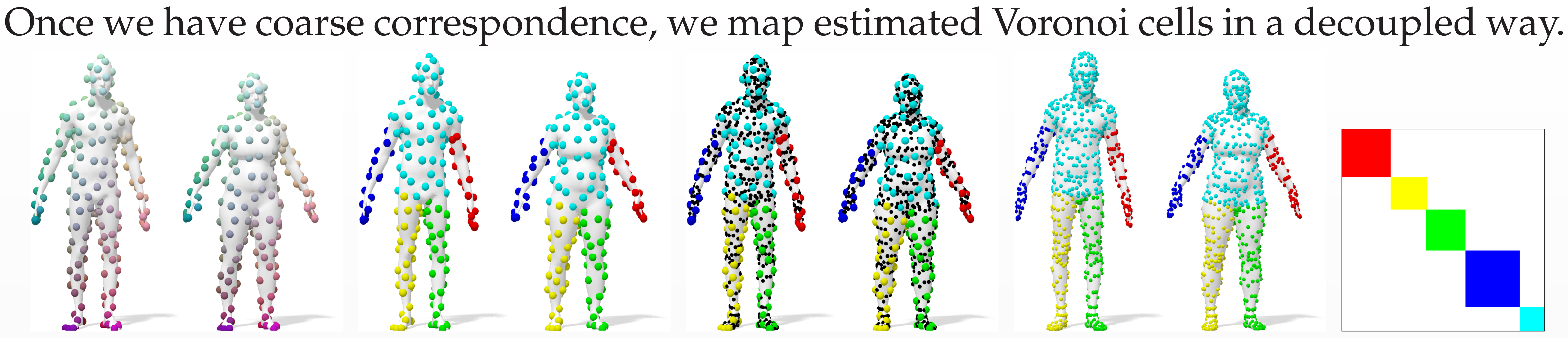
where $\nabla f(\mathbf{\Pi}) = \lambda\left(\mathbf{K}_{\mathcal{Y}}^\top \mathbf{\Pi} \mathbf{K}_{\mathcal{X}} + \mathbf{K}_{\mathcal{Y}} \mathbf{\Pi} \mathbf{K}_{\mathcal{X}}^\top\right) + \mathbf{F}_{\mathcal{Y}} \mathbf{F}_{\mathcal{X}}^\top.$



This algorithm can be interpreted as an *alternating diffusion* process [6].

Multiscale

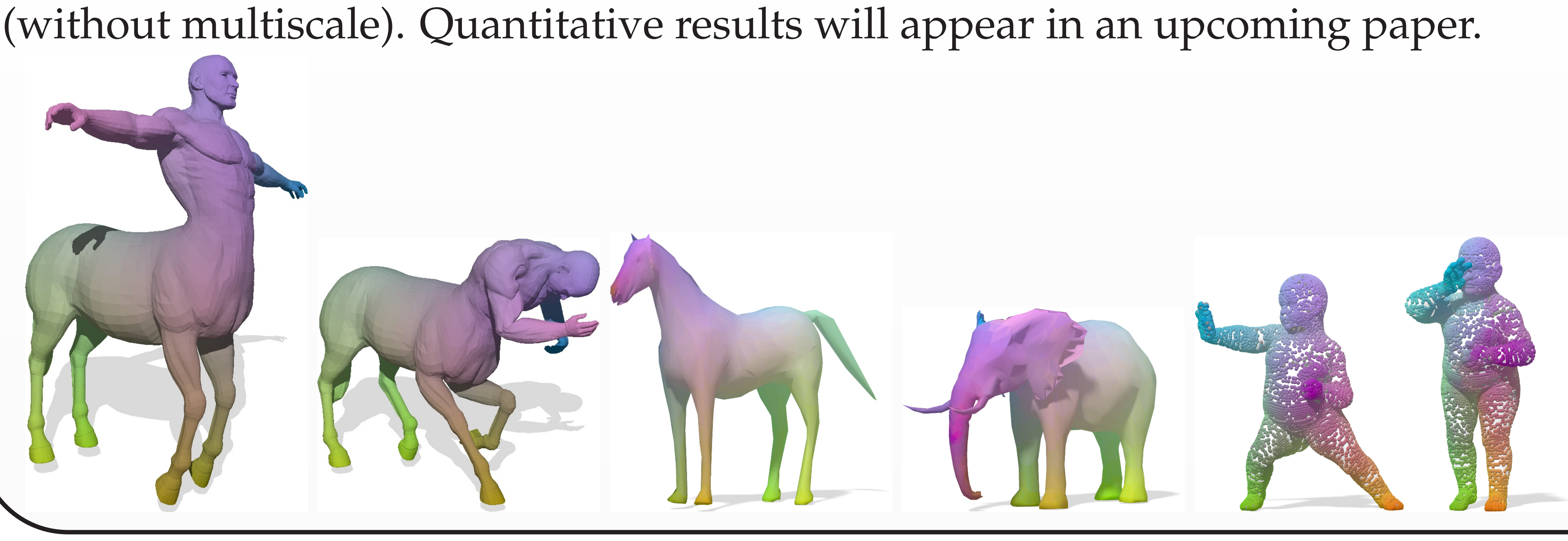
Once we have coarse correspondence, we map estimated Voronoi cells in a decoupled way.



(a) (b) (c) (d) (e)

Results

Matchings obtained using SHOT [1] and HKS [2] as pointwise descriptors and *heat kernels* as pairwise descriptors, between shapes with up to ~ 15000 vertices (without multiscale). Quantitative results will appear in an upcoming paper.



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