OPTIMAL NONLINEAR ESTIMATION OF PHOTON COORDINATES IN PET

Alexander Bronstein, Michael Bronstein, Michael Zibulevsky, Yehoshua Y. Zeevi
bron@aluf.technion.ac.il bron@tx.technion.ac.il mzib@ee.technion.ac.il zeevi@ee.technion.ac.il
Department of Electrical Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

ABSTRACT

We consider detection of high-energy photons in PET using thick scintillation crystals. Parallax effect and multiple Compton interactions in this type of crystals significantly reduce the accuracy of conventional detection methods. In order to estimate the scintillation point coordinates based on photomultiplier responses, we use asymptotically optimal nonlinear techniques, implemented by feed-forward neural networks, radial basis functions (RBF) networks, and neuro-fuzzy systems. Incorporation of information about angles of incidence of photons, significantly improves accuracy of estimation. The proposed estimators are fast enough to perform detection, using conventional computers.

1. INTRODUCTION

Detection of high-energy photons emitted as the result of positron decay is one of the most important low-level stages in PET imaging. In this paper we consider a detector based on the Anger scintillation camera [1]. Incident high-energy gamma quanta, generated due to positron decay produce scintillation effect in the crystal. As the result, a shower of low energy photons in the visible and UV spectra is emitted. These photons are collected by an array of photo-multipliers (PMTs), optically coupled to the scintillation crystal, and invoke electric impulses in them. The PMT responses are utilized in estimation of the scintillation point coordinates.

A non-collimated Anger camera, based on thick crystals with high photon penetration depth such as NaI(Tl), is considered in this work. Application of such thinck crystals in PET scanners is desirable, due to their low cost and very high light output; they were previously used primarily in gamma ray astronomy [5].

The majority of existing detection algorithms are based on centroid arithmetic, usually combined with correction maps [2]. Their application appears, however to be problematic in the case of thick crystals due to significant parallax observed at large radiation incidence angles.

Tomitani et al [11] proposed an iterative maximum likelihood algorithm for position estimation and depth encoding in thick scintillation crystals, in order to compensate for the parallax effect. However, an iterative approach necessitate extensive computations that prohibit real-time implementation.

Delorme et al [6] and Clément et al [4] have implemented artificial neural networks in a depth-encoding scintillation detection. The approach is flexible and offers advantages over iterative algorithms.

The depth-encoding approach leaves several problems open. First, the optimal tradeoff between planar and depth resolution. Second, multiple Compton interactions make the conception of "depth of interaction" ambiguous.

This work presents a solution for these problems, incorporating side information on the photon incidence angle into the process of position estimation. We use localized, asymptotically optimal, nonlinear estimators, implemented by feed-forward neural networks, radial basis functions (RBF) networks, and neuro-fuzzy systems. As a byproduct, we get accurate position estimation over the entire area of detector including the edges (similar phenomenon was observed by Mester and Zibulevsky [9] in SPECT). This is difficult to obtain with centroid arithmetics algorithms. We present a comparison of algorithms on a Monte Carlo simulation and discuss the prospects for practical implementation.

2. PARAMETRIC ESTIMATION USING NEURAL NETWORKS

Scintillation detector can be considered to be a complicated non-linear stochastic system that maps the photon line of flight (LOF) into a vector x of PMT responses. The stochastic aspects of this mapping are related to the random nature of the following factors:

- position of the fist interaction within the crystal
- possible multiple Compton interactions
- number of visible/UV photons in each interaction, registered by PMTs

Statistical effects of these factors depends heavily on the incidence angle, which can be estimated with reasonable accuracy using approximate LOF coordinates, measured by a pair of opposite detectors (obtained, for example, from the Anger algorithm).

Given the incidence angle, LOF is defined by planar coordinates $\boldsymbol{y}=(y_1,y_2)$ on the surface of the crystal. Therefore, for every incidence angle, one can implement an optimal nonlinear estimator of $\boldsymbol{y}$ of the form $\hat{\boldsymbol{y}}=\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})$, where $\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})$ is a family of functions, parameterized by the vector of parameters W.

A reasonable criterion for estimator optimality is the expectation of some error function $\mathbb{E}\left\{\varepsilon\left(\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})-\boldsymbol{y}\right)\right\}$, for example, the expected squared error $\mathbb{E}\left\{\left\|\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})-\boldsymbol{y}\right\|_2^2\right\}$.

We are interested in forms of $\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})$, that possess the property of a universal approximator; namely, when the number of parameters W is large enough, any bounded function $f(\boldsymbol{x})$ can be approximated with given accuracy over a bounded domain by an appropriate choice of W.

Given the PMT responses to a set of known LOFs $\left\{\boldsymbol{x}^i=\left(x_1^i,..,x_n^i\right),\boldsymbol{y}^i=f\left(\boldsymbol{x}^i\right)\right\}_{i=1}^N$ (referred to as a *training set*), we find such W, that minimizes the mean- squared error (MSE) on the training set, i.e:

$$\mathrm{W}=\underset{\mathrm{W}}{\operatorname{argmin}}\sum_{\boldsymbol{i}=1}^{\boldsymbol{N}}\left(\boldsymbol{\Phi}\left(\boldsymbol{x}^{\boldsymbol{i}};\mathrm{W}\right)-\boldsymbol{x}^{\boldsymbol{i}}\right)^2 .$$

This process is referred to as *training*. When the training set is sufficiently large, the MSE approximates the expected squared error with any desired accuracy. Under such conditions, a universal approximator $\boldsymbol{\Phi}(\boldsymbol{x};\mathrm{W})$ with sufficient parameters is capable of producing the optimal non-linear estimation.

In this work we used three type of universal approximators implemented as artificial neural networks (ANNs):

1. Multi-layer perceptron (MLP) [7]:

$$y_k^n=\sum_{i=1}^{N_{n-1}}\varphi\left(w_{k,n}^i y_i^{n-1}\right)+b_{k,n} \quad ; \quad y_k^0=x_k$$

$$\boldsymbol{\Phi}=\sum_{i=1}^{N_{L-1}}w_L^i y_i^{L-1}+b_L ,$$

where L is the number of layers; N_l is the number of neurons in each layer; y_k^n and $\left\{\boldsymbol{x}_{k,n},b_{k,n}\right\}$ are the output and the parameter vector of k-th neuron in n-th layer, respectively; $\boldsymbol{x}$ and $\boldsymbol{\Phi}$ are the network input and output, respectively; φ is some non-linear function, usually of a sigmoidal type.

2. Radial basis function (RBF) network [7]:

$$\boldsymbol{\Phi}=\sum_{k=1}^{N}w_k\beta\left(d\left(\boldsymbol{x},\boldsymbol{c}_k\right);\sigma_k\right)+b ,$$

where N is the number of neurons in the non-linear layer; $\left\{w,\boldsymbol{x}_k,b,\sigma_k\right\}$ are the network parameters; β is a Gaussian with controllable variance, σ_k, and mean, c_k.

3. Neuro-fuzzy system (NEFPROX) [10]. This method appears to be more efficient in both training and the complexity of the network itself. NEFPROX is used in this work along with MLP and RBF networks.

3. TWO-LEVEL SCHEME USING LOCALIZED ESTIMATORS

In this paper, we propose a more practical scheme based on a combination of coarse and fine estimators. The core of the detection algorithms is a set of fine estimators, implemented as neural networks. Fine estimators are trained on scintillation events in different (possibly overlapping) regions and at different incidence angles. Such a combination of estimators allows reduction in the size of each network and accelerates the training.

The sets of neural networks are trained independently on simulated (or on measured) PMT responses resulting from scintillation events in appropriate regions and angles. Incidence angle is estimated using additional information on the coincident event in the opposite detector.

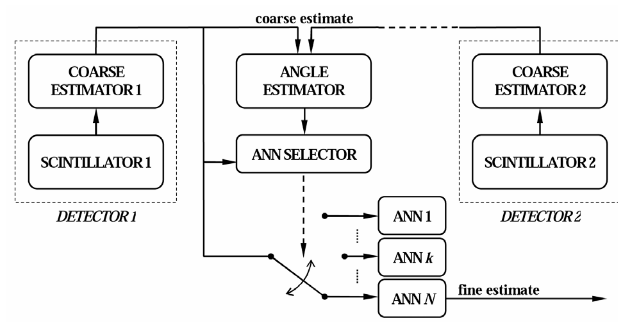

Fig. 1. Block diagram of a practical ANN-based photon detection algorithm: estimation of scintillation coordinates in detector 1 using side information from detector 2.

The use of ANNs also makes possible the calibration of the detection algorithm. The distortions of an Anger camera can be considered to be comprised of *characteristic* and *specific distortions*. Characteristic distortions are typical of a camera of a certain design, resulting from the detector geometry, scintillation crystal material and other factors related to the detector design. Specific distortions, on the contrary, are typical for a particular camera and may vary from camera to camera of the given design, resulting from manufacturing inaccuracies, the age of PMTs, etc.

Training the ANNs on a “characteristic” detector takes into consideration only the characteristic distortions and will probably provide mediocre results if applied “as is”, due to uncompensated specific distortions. Compensation for such imperfections is performed by tuning up the networks.

4. SIMULATIONS

In order to test the proposed approach and compare it with other algorithms, we performed a Monte Carlo simulation of ray tracing and gamma quanta interaction in a scintillation detector. The simulation was performed using a slightly modified version of TRIUMF detector modeling platform introduced by Tsang et al [8] in 1995.

A model of a NaI(Tl) scintillation crystal of size 210×210×45 *mm*, separated with a 20 *mm* glass light guide was simulated. The detector consisted of seven circular PMTs, each of radius 30 *mm*, with inter-tube gaps of 10 *mm*. The inter-tube area was assumed to consist of an ideal light-absorbing material.

Three tests were performed in order to analyze the effectiveness of different photon detection algorithms in so far as the effective detection region and the parallax effect. The effect of parallax was tested in a central region of the detector, for a large incidence angle (30°). A comparative test, with normal photons in a central region, was performed.

TABLE I

	Min.Error	Max.Error.	Avg.Error
Ideal Anger	4.9759	5.1251	5.0492
Linear regression	2.7248	5.2052	3.8609
MLP	2.8911	3.9501	3.4359
RBF	2.8994	3.7875	3.3093
NEFPROX	2.8478	4.4031	3.5772

Test I: Root mean-squared error (*mm*), incidence angle: 0º.

TABLE II

	Min.Error	Max.Error.	Avg.Error
Ideal Anger	7.1340	7.4158	7.2878
Linear regression	3.2015	6.3351	4.7618
MLP	3.3570	4.4614	3.8643
RBF	3.4071	4.4825	3.7918
NEFPROX	3.2437	5.0703	4.1138

Test II: Root mean-squared error (*mm*), incidence angle: 30º.

TABLE III

	Bias	RMS Err.	Std. Dev.	FWHM
Anger	-3.0364	4.3758	3.1511	2.79
Ideal Anger	—	4.9539	4.9539	4.40
MLP	0.4227	2.9998	2.9701	1.62

Parameters of error distribution at a single point, incidence angle: 0º. All values are given in *mm*.

TABLE IV

	Bias	RMS Err.	Std. Dev.	FWHM
Anger	-7.9219	9.1227	4.5243	6.52
Ideal Anger	—	7.1128	7.1128	10.25
MLP	0.9342	3.6721	3.5515	5.50

Parameters of error distribution at a single point, incidence angle: 30º. All values are given in *mm*.

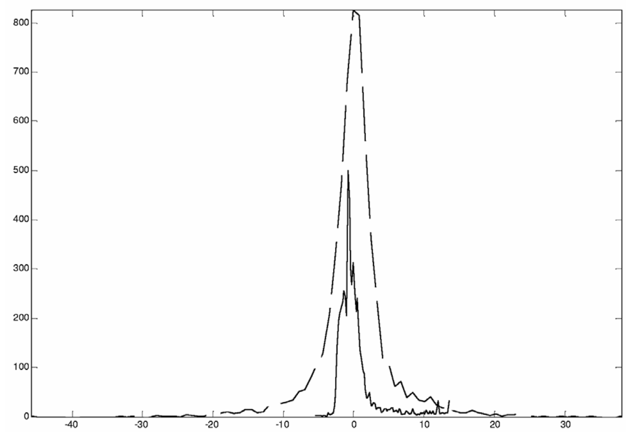

Fig. 2. Error histogram of ideally unbiased Anger algorithm (dashed) and MLP (solid) at a single point. Incidence angle: 0º. X-axis represents RMS error in *mm*.

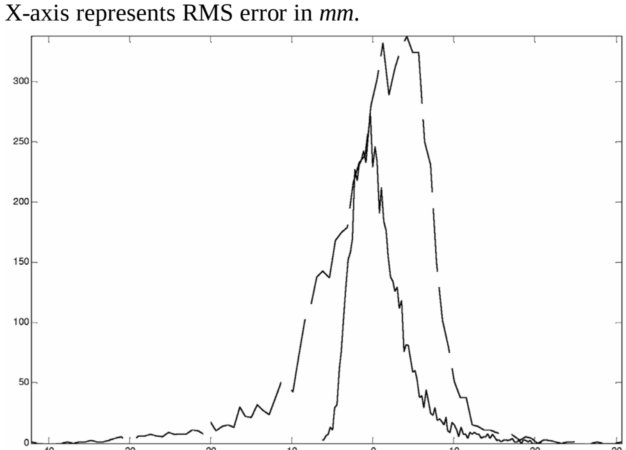

Fig. 3. Error histogram of ideally unbiased Anger algorithm (dashed) and MLP (solid) at a single point. Incidence angle: 30º. X-axis represents RMS error in *mm*.

In addition, utilization of the detector area and the influence of edge effects were tested in a distant region with normal photons [3].

The following algorithms were compared: the standard Anger algorithm with ideal unbiasing, local linear regression, MLP (with 15 neurons), RBF (with 50 kernels) and NEFPROX.

Tests were performed on sets of simulated data, using a 5×5 uniform grid and 1000 photons at each point. Training sets were constructed from simulated data on a 30×30 uniform grid in the appropriate region, where 100 photon incidences were simulated at each point of the grid in order to obtain reliable statistics. Energy discrimination, by removing about 25% of events below the photopeak from both training and test sets, was performed.

Tables I and II show the simulation results. X-axis resolution was used as the comparison criterion. Even an ideal, unbiased Anger's algorithm (unachievable in practice) appears the worst method in all tests. Local linear regression, which does not appear to be the best among the compared adaptive estimation algorithms, yields in all tests better results than Anger's algorithm. Non-linear estimation, using MLP and RBF networks, shows the best results in all tests. NEFPROX appeared less accurate, but resulted in significantly faster training and more computationally efficient network.

Tables III, IV and Figs. 2, 3 depict error distributions of Anger algorithm and MLP, compared at a single point on the detector. The MLP has significantly smaller bias and standard deviation, compared to the classical Anger algorithm. Our studies show that error distributions produced by ANN estimators tend to have smaller FWHM:standard deviation ratio compared to that of the Anger algorithm; this could result in additional image resolution improvement [3].

5. CONCLUSIONS

The proposed method of photon detection in PET, based on artificial neural networks, incorporates information about the incidence angle in the detection algorithm. This approach is capable of estimating directly the photon line of flight, given PMT responses from a pair of detectors. The proposed algorithm allows compensation for the parallax effect, it reduces the resolution degradation due to multiple Compton scattering and increased effective detection area. Our approach outperforms conventional detection algorithms in simulation studies.

In practice, a different version of the algorithm can be implemented. In particular, the neural networks can be trained over small regions of the detector on a range of angles with given angular resolution, and be fed with the angle as additional network input.

6. ACKNOWLEDGEMENTS

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